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(NASA-TT-F-12788) EFFECT ON THE HEAT TRANSFER PROCESS OF RADIATION AND ABSORPTION BY THE MEDIUM (TASS) 10 p

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Effect on the Heat Transfer Process of Radiation and Absorption by the Medium

L: P: FILLIPOV

Vestnik, Moscow University, vol. 1, No. 2, 1954, pp.51-57

Study of the question of the effect of the phenomena of radiation and absorption by the medium on the heat transfer process is of definite interest. The solution of this problem affords the possibility of greater understanding of the nature of the heat transfer process in regions of high temperature, and permits carrying out more exact computations of heat exchange. U. A. SURINOV [1] formulated certain general equations of heat radiation in the presence of an absorbing medium, however, he did not solve any specific problem. In this paper we try to carry the investigation of this problem to an end for the simplest case - the heat transfer through a plane layer.

Let us study the steady heat transfer through a layer formed by two solid walls. We will assume the walls to be reflecting mirrors and characterize their reflection coefficient by R, which can be considered, generally speaking, a function of the angle of incidence Θ and the frequency ν. Let the left wall of the layer $(x=0)$ have the temperature T_1 and the right $(x=L)$ the temperature T_2. Of interest to us is the temperature distribution in the layer and the magnitude of the heat flow through it.

The basic physical premises are:

1. The assumption that radiation and absorption in a volume element depend only on the properties and state of the medium in this volume. This permits the use of the concept of the coefficient of volume radiation e_ν, the

the absorption coefficient α_ν and the refraction index n_ν to describe the specified process. The relation between these three quantities is given by Kirchhoff's law:

$$\varepsilon_\nu = \frac{e_\nu}{\alpha_\nu n_\nu^2} \tag{1}$$

where ε_ν - the intensity of equilibrium radiation in a vacuum - is a general function of the frequency and the temperature.

2. It is assumed that the processes of molecular heat conduction and heat transfer are independent of the radiation, i.e., it is assumed that the heat flow through the medium is made up of the sum of the flow by radiation, Q_1, and the flow by molecular heat conduction, $Q_2 = -\lambda \frac{dt}{dx}$.

The first two premises permit the use of the transfer equation for radiation intensity:

$$\frac{dJ_\nu^+}{dx} = -\frac{\alpha_\nu}{\cos\theta}(J_\nu^+ - n_\nu^2 e_\nu) \tag{2}$$

$$\frac{dJ_\nu^-}{dx} = \frac{\alpha_\nu}{\cos\theta}(J_\nu^- - n_\nu^2 e_\nu) \tag{3}$$

Hence J_ν^+ is the absolute magnitude of the radiation intensity from the left wall to the right and J_ν^-, correspondingly, is from right to left. The angle θ varies between 0 and $\frac{\pi}{2}$ [2].

Taking into account that α_ν and $e_\nu n_\nu^2$ are functions of the temperature and therefore, of the x coordinate,

we write the general integrals of these equations:

$$(4)\qquad J_\nu^+ = J_\nu^+(0)\exp\left\{\frac{-1}{\cos\theta}\int_0^x \alpha_\nu\, d\eta\right\} + \frac{1}{\cos\theta}\int_0^x n_\nu^2 e_\nu \alpha_\nu \exp\left\{\frac{-1}{\cos\theta}\int_0^x \alpha_\nu\, d\eta\right\} d\xi$$

$$(5)\qquad J_\nu^- = J_\nu^-(L)\exp\left\{\frac{-1}{\cos\theta}\int_x^L \alpha_\nu\, d\eta\right\} + \frac{1}{\cos\theta}\int_x^L n_\nu^2 e_\nu \alpha_\nu \exp\left\{\frac{-1}{\cos\theta}\int_x^L \alpha_\nu\, d\eta\right\} d\xi$$

For simplicity, the index ν is omitted in later computations.

To find the value of the intensity on the boundaries, $J^+(0)$ and $J^-(L)$, we use the following relations:

$$(6)\qquad J^+(0) = \varepsilon_1(1 - R_1)n^2 + R_1 J^-(0)$$

$$(7)\qquad J^-(L) = \varepsilon_2(1 - R_2)n^2 + R_2 J^+(L)$$

Here, $\varepsilon_1 = \varepsilon(T_1)$ and $\varepsilon_2 = \varepsilon(T_2)$ are the radiating powers of absolutely black bodies at corresponding temperatures.

For $x = L$, (4), and for $x = 0$, (5), form, in conjunction with (6) and (7), a complete system of equations in the unknowns $J^-(0)$, $J^-(L)$, $J^+(0)$, $J^+(L)$. We obtain for the difference $(J^+ - J^-)$:

$$(8)\quad J^+ - J^- = \beta\Bigg\{1 - R_2\exp\left\{\frac{-2}{\cos\theta}\int_x^L \alpha\, d\eta\right\}\Bigg[\varepsilon_1 n^2(1-R_1)\exp\left\{\frac{-1}{\cos\theta}\int_0^x \alpha\, d\eta\right\}$$

$$+\frac{1}{\cos\theta}\int_0^x \varepsilon n^2\alpha\exp\left\{\frac{-1}{\cos\theta}\int_\xi^x \alpha\, d\eta\right\}d\xi$$

$$+\frac{R_1}{\cos\theta}\exp\left\{\frac{-1}{\cos\theta}\int_0^x \alpha\, d\eta\right\}\int\varepsilon n^2\alpha\exp\left\{\frac{-1}{\cos\theta}\int_0^\xi \alpha\, d\eta\right\}d\xi$$

$$-\Bigg[1 - R_1\exp\left\{\frac{-2}{\cos\theta}\int_0^x \alpha\, d\eta\right\}\varepsilon_2 n^2\exp\left\{\frac{-1}{\cos\theta}\int_x^L \alpha\, d\eta\right\}$$

$$+\frac{R_2}{\cos\theta}\exp\left\{\frac{-1}{\cos\theta}\int_x^L \alpha\, d\eta\right\}\int_x^L \varepsilon n^2\alpha\exp\left\{\frac{-1}{\cos\theta}\int_\xi^L \alpha\, d\eta\right\}d\xi$$

$$+\frac{1}{\cos\theta}\int_x^L \varepsilon n^2\alpha\exp\left\{\frac{-1}{\cos\theta}\int_x^L \alpha\, d\eta\right\}d\xi\Bigg\}$$

where β is the factor of repeated reflection, equal to

$$(9) \qquad \beta = \frac{1}{1 - R_1 R_2 \exp\left(\frac{-2}{\cos\theta} \int_0^L \alpha \, d\eta\right)}$$

The total heat flow equals

$$(10) \qquad Q = Q_1 + Q_2 = \iint (J_1^+ - J^-) \cos\theta \, dw \, d\nu - \lambda \frac{dt}{dx}$$

For steady flow the energy condition is

$$(11) \qquad \frac{dQ}{dx} \equiv 0$$

that is,

$$(12) \qquad \iint \frac{\partial (J^+ - J^-)}{\partial x} \cos\theta \, dw \, d\nu = \lambda \frac{d^2 t}{dx^2}$$

is the integro-differential equation in $t = t(x)$.

The form of the expression for $(J^+ - J^-)$ is so complex that it is impossible to study the question in the general case. Consequently, later, we limit ourselves to the following special assumptions:

1. We will consider the case of small temperature differences on the boundaries, therefore, making it possible to be limited to the first two terms in the series expansion of ε.

2. We assume that we can neglect the dependence of the absorption and refraction coefficients on the temperature.

3. For simplicity, we let $R_1 = R_2 = R$.

The listed assumptions facilitate the study without limiting the generality, sufficiently, and without limiting,

essentially, the applicability of the result to actual systems.

We can transform (12) into:

$$\frac{dy}{dx} = \int_0^L K(\xi,x)\, y(\xi)\, d\xi \tag{13}$$

where

$$y = \frac{dt}{dx} \tag{14}$$

$$K(\xi,x) = \begin{cases} K_1(\xi,x) & 0 \leq \xi \leq x \\ K_2(\xi,x) & x \leq \xi \leq L \end{cases} \tag{15}$$

and

$$K_1 = \frac{2\pi}{\lambda} \int_0^\infty d\nu\, \alpha \frac{\partial \varepsilon}{\partial t}\, n^2 \int_1^\infty \frac{\beta}{s^3} \left[1+Re^{-2(L-x)\alpha s}\right]\left[e^{-(x-\xi)\alpha s} - Re^{-(x+\xi)\alpha s}\right] ds \tag{16}$$

$$K_2 = -\frac{2\pi}{\lambda} \int_0^\infty d\nu\, \alpha \frac{\partial \varepsilon}{\partial t}\, n^2 \int_1^\infty \frac{\beta}{s^3} \left[1+Re^{-2\alpha x s}\right]\left[e^{-(\xi-x)\alpha s} - Re^{-(2L-\xi-x)\alpha s}\right] ds \tag{17}$$

$$\beta = \frac{1}{1 - R^2 \exp(-2L\alpha s)} \tag{18}$$

It is seen that for $\frac{1}{\lambda} \longrightarrow 0$ the temperature distribution becomes linear. This permits using the expansion of (13) in terms of $\frac{1}{\lambda}$ to study the question in cases when the temperature distribution differs slightly from the linear.

Let us put

$$t - T_1 = \frac{\Delta T}{L} x + \frac{1}{\lambda} \varphi(x) \tag{19}$$

$\Delta T = T_2 - T_1$, neglecting later terms of the expansion. In this case the solution of (13) is:

$$t - T_1 = \frac{\Delta T}{L}\left[x + \psi(x) - \left(1 - \frac{2x}{L}\right)\psi(0)\right] \tag{20}$$

where

$$(21)\qquad \psi(x) = \frac{2\pi}{\lambda}\int_0^\infty d\nu \frac{\partial\varepsilon}{\partial t} n^3 \int_1^\infty \frac{(1+R)^2}{\alpha^3 s^3}\beta\left\{e^{-(L-x)\alpha s}\left[1+Re^{-2\alpha sx}\right] - e^{-x\alpha s}\left[1+Re^{-2(L-x)\alpha s}\right]\right\} ds$$

To consider the case of a weakly-absorbing medium, we use the method of expanding the integrand in powers of α. We obtain

$$(22)\qquad \theta = \tau + \tau(\tau-1)(2\tau-1)\,D$$

where

$$(23)\qquad \theta = \frac{t-T_1}{T_2-T_1}\ ;\ \tau = \frac{x}{L}\ ;\ D = \frac{L^2\pi}{3\lambda}\int_0^\infty \alpha \frac{\partial\varepsilon}{\partial t}\,d\nu$$

The curve $\theta = \theta(\tau)$ is shown in figure 1.

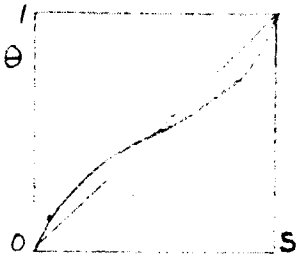

Figure 1

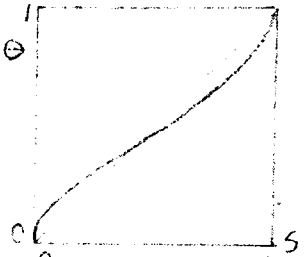

Figure 2

The maximum deflection from linearity occurs at a distance of about $\frac{1}{5}$ from the wall and is, for example, 0.1 D. Therefore, a practical criterion of the smallness of the deflection of the temperature distribution from linearity can be, in this case:

$$(24)\qquad \delta = \frac{L^2}{10\lambda_0}\int_0^\infty \alpha \frac{\partial\varepsilon}{\partial t}\,d\nu$$

It is interesting that the temperature distribution appears to be independent of the quality of the wall in this case.

In the other limiting case for a strongly-absorbing medium we obtain:

$$(25)\quad \frac{dt}{dx} = \frac{\Delta T}{L}\left\{1+\frac{2\pi}{\lambda}\int_0^\infty d\nu \frac{\partial \varepsilon}{\partial t} n^3 \int_1^\infty \frac{(1+R)}{\alpha s^4}\left[e^{-x\alpha s}+e^{-(L-x)\alpha s}-\frac{2}{\alpha L s}\right] ds\right\}$$

The temperature distribution is shown in figure 2.

To find the magnitudes of the heat flow, we use (10) and (20) discarding terms of order $\frac{1}{\lambda}$. We obtain:

$$(26)\quad q = -\frac{\lambda \Delta t}{L} = \frac{4\pi}{3}\frac{\Delta T}{L}\int_0^\infty \frac{n^2}{\alpha}\frac{\partial \varepsilon}{\partial t} d\nu + \frac{4\pi \Delta T}{L^2}\int_0^\infty d\nu \frac{\partial \varepsilon}{\partial t}\frac{n^3}{\alpha^2}\int_1^\infty \frac{(1-e^{-L\alpha s})(1+R)}{1+Re^{-L\alpha s}}\frac{ds}{s^3}$$

For weakly-absorbing media, this formula becomes:

$$(27)\quad q = -\frac{\lambda \Delta T}{L} - 2\pi \Delta T \int_0^\infty \frac{\partial \varepsilon}{\partial t} d\nu \int_1^\infty \frac{1-R}{1+R}\frac{1}{s^3} ds - \frac{\Delta T L}{3} 2\pi \int_0^\infty \frac{\partial \varepsilon}{\partial t} \alpha\, d\nu \int_1^\infty \frac{1-4R+R^2}{(1+R)^3}\frac{ds}{s^2}$$

The first term of this expression gives the heat flow by molecular heat conduction, the second describes the effect of wall radiation, the third is the desired heat flow variation at the expense of the radiation and absorption processes by the radiant energy of the medium. The maximum value of this terms equals $2\Delta T L \int_0^\infty \frac{\partial \varepsilon}{\partial t}\alpha\, d\nu$ As upper limit for the magnitude of relative heat-flow variation, we obtain

$$(28)\quad \frac{q}{q} = \frac{2L^2}{\lambda}\frac{\partial}{\partial t}\int_0^\infty \alpha \varepsilon\, d\nu$$

Therefore, the studied effect, in the case of weakly-

absorbing media, influences the heat flow twenty times more greatly than the temperature distribution.

In the case of strongly-absorbing media, we obtain from (26):

$$q = -\frac{\lambda \Delta T}{L} - \frac{4\pi}{3}\frac{\Delta T}{L}\int_0^\infty \frac{n^2}{\alpha}\frac{\partial \varepsilon}{\partial t}\,d\nu \tag{29}$$

Therefore the heat conduction coefficient is increased, here, by

$$\lambda^* = \frac{4\pi}{3}\int_0^\infty \frac{n^2}{\alpha}\frac{\partial \varepsilon}{\partial t}\,d\nu \tag{30}$$

independently of the properties of the wall. For high temperatures, it must be sufficiently large, while the temperature distribution will differ slightly from the linear, thanks to the exponential factor in (25).

Let us use the formulas obtained and calculate the influence of the considered effect in the case of heat transfer through a weakly-absorbing medium- gases.

We use (28) for the computations. Let us replace the integral in (28) by the following expression:

$$\int_0^\infty \alpha_\nu \varepsilon_\nu\,d\nu = \frac{B\sigma T^4}{2\pi} \tag{31}$$

where p is the pressure, σ is the Stefan-Boltzmann constant, B is the limit of the ratio of the degree of blackness for a plane layer of a gas to the product pL as the latter goes to zero. To differentiate (31) with respect to the temperature, we neglect the dependence of the temperature on the degree of blackness by comparison

with the fourth power of the temperature. Finally we obtain:

$$(32) \qquad \frac{q}{q} \simeq \frac{4}{3}\sigma\frac{B_\nu L^3 T^3}{\lambda}$$

This formula can be used for the practical evaluation of the limits of influence of the considered effects. Thus, for CO_2 at room temperature and 1 atmos. pressure, we obtain $\frac{\Delta}{q} \simeq 5\cdot10^{-3}\ L^2/cm^2$. Therefore, the heat flow variation at the expense of the radiation of the medium can become noticeable, in this case, at a distance of a few centimeters. For a pressure of 100 atmos. this effect is felt at a distance of a few millimeters. The influence of the temperature is still greater. For water vapor, similar orders of magnitude are obtained.

Prof. of Molecular
and Heat Phenomena Oct. 1952

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